Interpretation of a Log-Log Model

1 Log-Log Model Overview

A log-log model takes the form:

$$\log(Y) = \beta_0 + \beta_1 \log(X) + \varepsilon \tag{1}$$

where both the dependent variable Y and the independent variable X are log-transformed. The coefficient β_1 in this model represents the **elasticity** of Y with respect to X:

$$\beta_1 = \frac{\%\text{change in } Y}{\%\text{change in } X} \tag{2}$$

This means that a 1% increase in X leads to a β_1 percent increase in Y, holding other factors constant.

2 Example

Suppose we estimate the following regression, where *Sales* and *Advertising* are both measured :

$$\log(Sales) = 2.5 + 0.8 \log(Advertising) \tag{3}$$

Here, the coefficient 0.8 means that a **1%** increase in Advertising expenditure leads to a **0.8%** increase in Sales.
Suppose Advertising expenditure increases from 10 to 11 units (a **10% increase**). Using the elasticity concept, Sales will increase by:

$$0.8 \times 10\% = 8\% \tag{4}$$

If initial Sales at **Advertising = 10** is S, the new Sales at **Advertising = 11** will be approximately $1.08S$.
If we need to predict actual values, we exponentiate both sides:

$$Sales = e^{2.5} \cdot Advertising^{0.8} \tag{5}$$

This transformation allows us to recover Sales in its original unit. The value $e^{2.5} \approx 12.18$ represents the **baseline Sales** when $Advertising = 1$ (assuming the units are meaningful in the given context). This means that if Advertising expenditure is **1 unit**, the expected Sales will be **12.18 units**.
In most cases, there is no need to apply an exponential transformation for a log-log model, as the interpretation in terms of elasticity is probably even more intuitive.

3 Key Takeaways

- The coefficient 0.8 tells us the percentage change in Sales for a percentage change in Advertising.
- e^{β_0} provides a scale factor for Sales when Advertising is at **1 unit**.
- No need for further exponentiation to interpret elasticity, as it's already in percentage terms.