Interpreting Large Coefficients in Regressions with Log-Dependent Variable

1 Problem from Large Coefficients

When estimating a regression of the form:

$$\ln(Y) = \beta X + \varepsilon, \tag{1}$$

the coefficient β is often interpreted as the approximate percentage change in Y for a one-unit change in X, given by:

$$\%\Delta Y \approx 100 \times \beta. \tag{2}$$

However, this is only an approximation. The exact percentage change is given by:

$$\%\Delta Y = (e^{\beta} - 1) \times 100. \tag{3}$$

When β is small (e.g., 0.05), the approximation $100 \times \beta$ is close to the exact percentage change.
When β is large (e.g., 1 or more), the approximation underestimates the actual effect.
For example:

- If $\beta = 0.05$, then:
$$e^{0.05} - 1 \approx 5.13\%, \tag{4}$$
which is close to 5%.
- If $\beta = 1$, then:
$$e^{1} - 1 \approx 172\%, \tag{5}$$
which is far from the approximation of 100%.

2 When is the Approximation Accurate?

The approximation works well for small values of $|\beta|$, but it becomes increasingly inaccurate as $|\beta|$ grows. The following rules of thumb apply:

- If $|\beta| < 0.1$, the approximation is very accurate (error $< 0.5\%$).
- If $|\beta|$ is between 0.1 and 0.2, the approximation is reasonable (error $< 2\%$).
- If $|\beta| > 0.3$, the approximation starts to diverge significantly.
- If $|\beta| \geq 0.5$, the approximation underestimates the true percentage change, and the exact formula should be used.
- If $|\beta| \geq 1$, the approximation is highly inaccurate and should never be used.

3 Comparison of Approximation and Exact Calculation

Below is a comparison of the two methods for different values of β:

β	Approximation (100β)	Exact Calculation ($(e^{\beta} - 1) \times 100$)
0.05	5%	5.13%
0.1	10%	10.52%
0.2	20%	22.14%
0.5	50%	64.87%
1.0	100%	171.83%
1.5	150%	348.85%

4 Conclusion

For small values of β, the approximation 100β is acceptable. However, for larger coefficients (especially above 0.3), the exact formula should be used to avoid underestimating the percentage change in Y.